UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/11**

Paper 1 **October/November 2013**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

0581709578

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages.

UNIVERSITY of CAMBRIDGE International Examinations

[Turn over

1 The curve C has polar equation $r = 2e^{\theta}$, for $\frac{1}{6}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$. Find

(i) the area of the region bounded by the half-lines $\theta = \frac{1}{6}\pi$, $\theta = \frac{1}{2}\pi$ and C, [2]

(ii) the length of C. [3]

2 The cubic equation $x^3 - px - q = 0$, where p and q are constants, has roots α, β, γ. Show that

(i) $\alpha^2 + \beta^2 + \gamma^2 = 2p$, [1]

(ii) $\alpha^3 + \beta^3 + \gamma^3 = 3q$, [2]

(iii) $6(\alpha^5 + \beta^5 + \gamma^5) = 5(\alpha^3 + \beta^3 + \gamma^3)(\alpha^2 + \beta^2 + \gamma^2)$. [3]

3 It is given that

$$S_n = \sum_{r=1}^{n} u_r = 2n^2 + n.$$

Write down the values of S_1, S_2, S_3, S_4. Express u_r in terms of r, justifying your answer. [4]

Find

$$\sum_{r=n+1}^{2n} u_r.$$ [3]

4 It is given that

$$I_n = \int_0^1 \frac{x^n}{\sqrt{(1+2x)}}\,dx.$$

Show that, for $n \geqslant 1$,

$$(2n+1)I_n = \sqrt{3} - nI_{n-1}.$$ [3]

Show that

$$I_3 = \tfrac{2}{35}(\sqrt{3} + 1).$$ [4]

5 It is given that $y = (1+x)^2 \ln(1+x)$. Find $\dfrac{d^3y}{dx^3}$. [3]

Prove by mathematical induction that, for every integer $n \geqslant 3$,

$$\frac{d^ny}{dx^n} = (-1)^{n-1}\frac{2(n-3)!}{(1+x)^{n-2}}.$$ [5]

6 The linear transformation $\mathrm{T} : \mathbb{R}^4 \to \mathbb{R}^4$ is represented by the matrix $\mathbf{M}$, where

$$\mathbf{M} = \begin{pmatrix} 1 & -3 & -1 & 2 \\ 4 & -10 & 0 & 2 \\ 1 & -1 & 3 & -4 \\ 5 & -12 & 1 & 1 \end{pmatrix}.$$

Find, in either order, the rank of $\mathbf{M}$ and a basis for the null space K of T. [6]

Evaluate

$$\mathbf{M}\begin{pmatrix} 1 \\ -2 \\ -3 \\ -4 \end{pmatrix},$$

and hence show that every solution of

$$\mathbf{Mx} = \begin{pmatrix} 2 \\ 16 \\ 10 \\ 22 \end{pmatrix}$$

has the form

$$\mathbf{x} = \begin{pmatrix} 1 \\ -2 \\ -3 \\ -4 \end{pmatrix} + \lambda\mathbf{e}_1 + \mu\mathbf{e}_2,$$

where λ and μ are real numbers and $\{\mathbf{e}_1, \mathbf{e}_2\}$ is a basis for K. [3]

7 The square matrix $\mathbf{A}$ has λ as an eigenvalue with $\mathbf{e}$ as a corresponding eigenvector. Show that $\mathbf{e}$ is an eigenvector of $\mathbf{A}^2$ and state the corresponding eigenvalue. [3]

Find the eigenvalues of the matrix $\mathbf{B}$, where

$$\mathbf{B} = \begin{pmatrix} 1 & 3 & 0 \\ 2 & 0 & 2 \\ 1 & 1 & 2 \end{pmatrix}.$$ [4]

Find the eigenvalues of $\mathbf{B}^4 + 2\mathbf{B}^2 + 3\mathbf{I}$, where $\mathbf{I}$ is the 3×3 identity matrix. [3]

8 The plane Π_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$. Find a cartesian equation of Π_1. [3]

The plane Π_2 has equation $2x - y + z = 10$. Find the acute angle between Π_1 and Π_2. [2]

Find an equation of the line of intersection of Π_1 and Π_2, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$. [5]

9 The curve C has parametric equations

$$x = t^2, \quad y = t - \tfrac{1}{3}t^3, \quad \text{for } 0 \leqslant t \leqslant 1.$$

Find the surface area generated when C is rotated through 2π radians about the x-axis. [6]

Find the coordinates of the centroid of the region bounded by C, the x-axis and the line $x = 1$. [6]

10 The curve C has equation

$$y = \frac{px^2 + 4x + 1}{x + 1},$$

where p is a positive constant and $p \neq 3$.

(i) Obtain the equations of the asymptotes of C. [3]

(ii) Find the value of p for which the x-axis is a tangent to C, and sketch C in this case. [4]

(iii) For the case $p = 1$, show that C has no turning points, and sketch C, giving the exact coordinates of the points of intersection of C with the x-axis. [5]

11 Answer only **one** of the following two alternatives.

EITHER

State the fifth roots of unity in the form $\cos\theta + \mathrm{i}\sin\theta$, where $-\pi < \theta \leqslant \pi$. [2]

Simplify

$$\left(x - [\cos\tfrac{2}{5}\pi + \mathrm{i}\sin\tfrac{2}{5}\pi]\right)\left(x - [\cos\tfrac{2}{5}\pi - \mathrm{i}\sin\tfrac{2}{5}\pi]\right).$$ [2]

Hence find the real factors of

$$x^5 - 1.$$ [2]

Express the six roots of the equation

$$x^6 - x^3 + 1 = 0$$

as three conjugate pairs, in the form $\cos\theta \pm \mathrm{i}\sin\theta$. [6]

Hence find the real factors of

$$x^6 - x^3 + 1.$$ [2]

OR

Given that

$$y^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 6y^2\frac{\mathrm{d}y}{\mathrm{d}x} + 2y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + 3y^3 = 25\mathrm{e}^{-2x}$$

and that $v = y^3$, show that

$$\frac{\mathrm{d}^2v}{\mathrm{d}x^2} - 6\frac{\mathrm{d}v}{\mathrm{d}x} + 9v = 75\mathrm{e}^{-2x}.$$ [4]

Find the particular solution for y in terms of x, given that when $x = 0$, $y = 2$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$. [10]